Calculator Lab: Numerical Integration I

In this lab we will find numerical approximations to the following definite integrals:

$$\int_1^4 \sqrt{x}dx \qquad \text{and} \qquad \int_0^1 e^{-x^2}dx$$

(Ignore the fact that the first integral can be done easily by the Fundamental Theorem of Calculus, and that your calculator has a built-in feature to calculate both integrals.)

You will want to use the `ALLSUMS` program on your calculator. The programs are available at the Calculus Webpage if you follow the *Calculator Programs* link under *Other Links*. (See http://math.arizona.edu/~krawczyk/calcul.html)

Background

Recall how we construct the definite integral, $\int_a^b f(x)dx$ (which refers to the area under the curve $f(x)$ from a to b). Divide the interval from a to b into n equal subdivisions, and call the width of an individual subdivision Δx, so $\Delta x = \frac{b-a}{n}$. Let $x_0, x_1, x_2, \ldots, x_n$ be the endpoints of the subdivisions. We construct the sums:

$$\text{Left-hand Sum} = \text{LEFT}(n) = f(t_0)\Delta t + f(t_1)\Delta t + \cdots + f(t_{n-1})\Delta t = \sum_{i=0}^{n-1} f(t_i)\Delta t$$

and

$$\text{Right-hand Sum} = \text{RIGHT}(n) = f(t_1)\Delta t + f(t_2)\Delta t + \cdots + f(t_n)\Delta t = \sum_{i=1}^{n} f(t_i)\Delta t.$$

Midpoint Rule. Here we use the midpoint of each interval to determine the height of each rectangle in the Riemann sum.

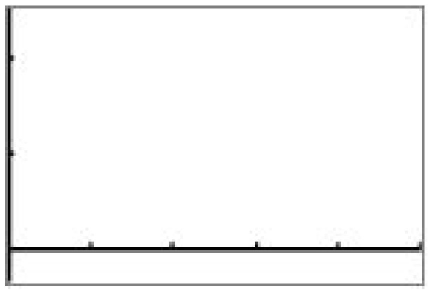
$$\begin{aligned} \text{MID}(n) &= f\left(\frac{t_0+t_1}{2}\right)\Delta t + f\left(\frac{t_1+t_2}{2}\right)\Delta t + \cdots + f\left(\frac{t_{n-1}+t_n}{2}\right)\Delta t \\ &= \sum_{i=0}^{n-1} f\left(\frac{t_i+t_{i+1}}{2}\right)\Delta t \end{aligned}$$

Trapezoid Rule. Here we use trapezoids to approximate the area in each subdivision instead of rectangles. Thus the trapezoid rule is the average of the left-hand and right-hand sums.

$$\begin{aligned} \text{TRAP}(n) &= \frac{f(t_0)+f(t_1)}{2}\Delta t + \frac{f(t_1)+f(t_2)}{2}\Delta t + \cdots + \frac{f(t_{n-1})+f(t_n)}{2}\Delta t \\ &= \sum_{i=0}^{n-1} \frac{f(t_i)+f(t_{i+1})}{2}\Delta t \\ &= \frac{\text{LEFT}(n)+\text{RIGHT}(n)}{2} \end{aligned}$$

Part I: Left and Right Hand Sums

1. (a) Sketch a graph of $\sqrt{x}$ with `xMin` = 0, `xMax` = 5 `yMin` = -0.3, `yMax` = 2.5 in the box on the left. Sketch a graph of e^{-x^2} with `xMin` = -0.5, `xMax` = 1.5 `yMin` = -0.3, `yMax` = 1.2 in the box on the right.

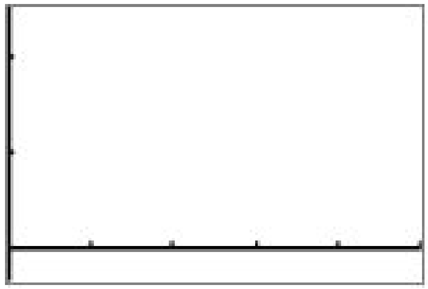


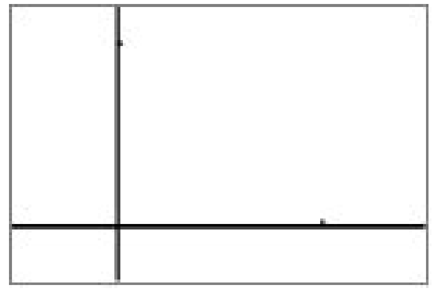
(b) In the above graphs, draw the left hand sum approximation of $\int_1^4 \sqrt{x}dx$ and $\int_0^1 e^{-x^2} dx$ with 2 subdivisions (referred to as LEFT(2)).

(c) Will LEFT(2) be an upper or a lower bound for the actual values of $\int_1^4 \sqrt{x}dx$ and $\int_0^1 e^{-x^2} dx$?

(d) Compute by hand LEFT(2) for $\int_1^4 \sqrt{x}dx$ and $\int_0^1 e^{-x^2} dx$.

2. (a) Sketch the graphs of $\sqrt{x}$ and e^{-x^2} from the previous page.





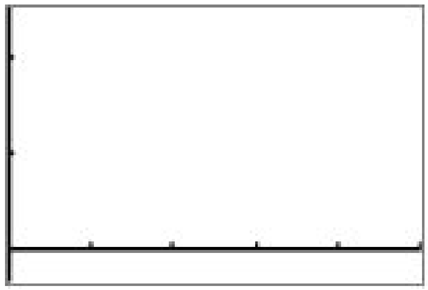
(b) In the above graphs, draw the right hand sum approximation of $\int_1^4 \sqrt{x}dx$ and $\int_0^1 e^{-x^2}dx$ with 2 subdivisions (referred to as RIGHT(2)).

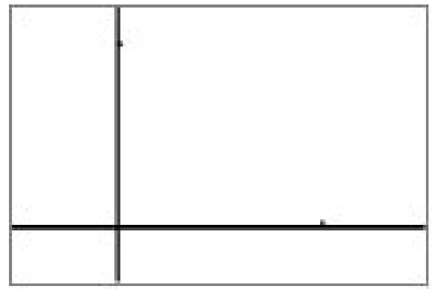
(c) Will RIGHT(2) be an upper or a lower bound for the actual values of $\int_1^4 \sqrt{x}dx$ and $\int_0^1 e^{-x^2}dx$?

(d) Compute by hand RIGHT(2) for $\int_1^4 \sqrt{x}dx$ and $\int_0^1 e^{-x^2}dx$.

Part II: Midpoint and Trapezoid Rules

3. (a) Sketch the graphs of $\sqrt{x}$ and e^{-x^2} from the previous pages.



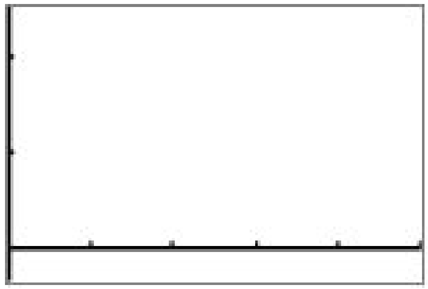


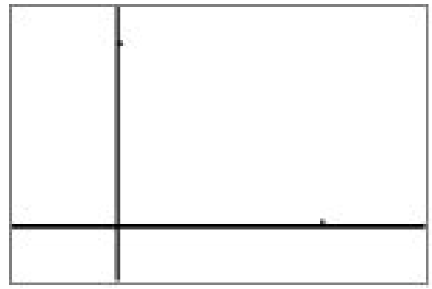
(b) In the above graphs, draw the midpoint rule approximation of $\int_1^4 \sqrt{x}dx$ and $\int_0^1 e^{-x^2} dx$ with 2 subdivisions (referred to as MID(2)).

(c) Will MID(2) be an upper or a lower bound for the actual values of $\int_1^4 \sqrt{x}dx$ and $\int_0^1 e^{-x^2} dx$?

(d) Compute by hand MID(2) for $\int_1^4 \sqrt{x}dx$ and $\int_0^1 e^{-x^2} dx$.

4. (a) Sketch the graphs of $\sqrt{x}$ and e^{-x^2} from the previous pages.





(b) In the above graphs, draw the trapezoid rule approximation of $\int_1^4 \sqrt{x}dx$ and $\int_0^1 e^{-x^2}dx$ with 2 subdivisions (referred to as TRAP(2)).

(c) Will TRAP(2) be an upper or a lower bound for the actual values of $\int_1^4 \sqrt{x}dx$ and $\int_0^1 e^{-x^2}dx$?

(d) Compute by hand TRAP(2) for $\int_1^4 \sqrt{x}dx$ and $\int_0^1 e^{-x^2}dx$.

Part III: Wrap-up

5. Using the `ALLSUMS` program on your calculator, fill in the following charts:

Approximations of $\int_1^4 \sqrt{x}dx$

n	LEFT(n)	RIGHT(n)	MID(n)	TRAP(n)
10				
50				
100				
250				

Approximations of $\int_0^1 e^{-x^2}dx$

n	LEFT(n)	RIGHT(n)	MID(n)	TRAP(n)
10				
50				
100				
250				

6. (a) When will LEFT(n) be an upper bound for $\int_a^b f(x)dx$? When will it be a lower bound?

(b) When will RIGHT(n) be an upper bound for $\int_a^b f(x)dx$? When will it be a lower bound?

(c) When will MID(n) be an upper bound for $\int_a^b f(x)dx$? When will it be a lower bound?

(d) When will TRAP(n) be an upper bound for $\int_a^b f(x)dx$? When will it be a lower bound?